Basic Sturm–Liouville problems

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Corrigendum

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This mistake is minor and it by no means affects the results of the paper. It is concerned with lemma 2.1 which states that 'The necessary and sufficient condition for the existence of the q-integral $\int_0^x f(t)\, d_q t$ is that $\lim_{k\to\infty} xq^k f(xq^k) = 0$.' In a private communication, Professor George Gasper pointed out that this is not correct and he gave the counterexample $f(x) = \frac{1}{x \ln x}$, $0 < x < 1$. We found that this condition, which is obviously necessary, is not sufficient and the statement

$$\lim_{k\to\infty} xq^k f(xq^k) = 0 \iff \exists \alpha \in [0, 1[\ \exists C > 0,\ |f(xq^k)| \leqslant C|xq^k|^{-\alpha},\ k \in \mathbb{N}$$

is not true. However this mistake will not affect any of the results of the paper. It has been used only at the beginning of the proof of theorem 5.1 to prove that $D_q c_i$, $i = 1, 2$, are in $L_q^1(0, a)$. To correct the situation, we redefine the set A_f and consequently the part of the proof on page 3788 from line 6 to line 12 should be replaced by:

Define the q-geometric set A_f by

$$A_f := x \in [0, a] : \quad \int_0^x |f(t)|^2\, d_q t \text{ exists.} \tag{5.7}$$

A_f is a q-geometric set containing 0, aq^m, $m \in \mathbb{N}$ *since* $f \in L_q^2(0, a)$. *From the Hölder inequality we obtain*

$$\int_0^x |\theta_i(qt, \lambda) f(qt)|\, d_q t \leqslant \left(\int_0^x |\theta_i(qt, \lambda)|^2\, d_q t\right)^{1/2} \left(\int_0^x |f(t)|^2\, d_q t\right)^{1/2}, \quad x \in A_f. \tag{5.8}$$

Since f, $\theta_i \in L_q^2(0, a)$, $i = 1, 2$ *and* $\Delta(\lambda) \neq 0$, *then* $D_q c_i(\cdot)$, $i = 1, 2$, *are q-integrable on* $[0, x]$ *for all* $x \in A_f$. *Hence direct computations lead to the following appropriate solutions of* (5.6).

The proof then continues as in the paper. It should also be noted that the definition of the operator $\mathcal{G}$ (page 3791, equation (6.6)) remains true. In fact even if we assume that f is q-regular at zero, it makes no restriction on the results since after constructing $G(x, \xi)$, the q-integral operator $\mathcal{G}$ is well defined on the entire $L_q^2(0, a)$.

We would also like to correct two misprints. First, in equation (2.6) $(x(1-q))^{1/2}$ should be replaced by $\left(xq^{-1/2}(1-q)\right)^{1/2}$, and on page 3793, line 5 from the bottom, $\sin(\sqrt{\lambda_n}; q)$ should be $\sin(\sqrt{\lambda_n} x; q)$.

The authors thank Professor George Gasper for his comment and apologize for any inconvenience.